Integration

Properties of Integrals

- $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int kf(x)\,dx = k\int f(x)\,dx$, where k is a real number.

Forgetting to include the constant '$+c$' after integrating or including 'dx' when integrating.

Rules for Anti-Differentiation

- $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$, where $n \neq -1$
- $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \log_e(x) + c$
- $\int \frac{1}{(ax+b)}dx = \int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e(ax+b) + c$
- $\int e^x\,dx = e^x + c$
- $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + c$
- $\int \sin(ax)\,dx = \frac{-1}{a}\cos(ax) + c$
- $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$
- $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
- $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

Note: The antidifferentiation rules described ONLY apply if the expression inside the brackets is linear.

If the expression is of any other kind, you may need to expand it if possible before integrating, or you must use your CAS calculator to integrate the function.

Find the antiderivative of $\frac{3}{\sqrt{x}} - \frac{1}{(2x-2)}$

$$\int \frac{3}{\sqrt{x}} - \frac{1}{(2x-2)}\,dx$$

$$= \int 3x^{\frac{-1}{2}} - \frac{1}{(2x-2)}\,dx$$

$$= \frac{3}{\frac{1}{2}}x^{\frac{1}{2}} - \frac{1}{2}log_e(2x-2) + c$$

$$= 6x^{\frac{1}{2}} - \frac{1}{2}log_e(2x-2) + c$$

$$= 6\sqrt{x} - \frac{1}{2}log_e(2x-2) + c$$

Fundamental Theorem of Calculus

If f is a continuous function on the interval [a, b] then

$$\int_a^b f(x)\,dx = [F(x)]_a^b$$

$$= F(b) - F(a)$$

Where F is the antiderivative of f.

Properties of Definite Integrals

- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
- $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, providing $a < c < b$