Statistics

Confidence Intervals

An approximate confidence interval for a population proportion is given by

$$\left(\hat{p} - z\sigma_{\hat{p}}, \hat{p} + z\sigma_{\hat{p}}\right)$$

where

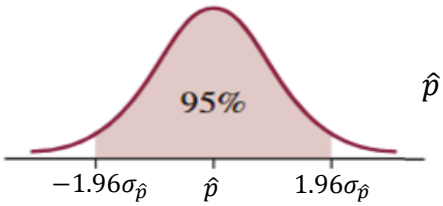
$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The confidence interval (CI) is a range of values that's likely to include a population value with a certain degree of confidence.

For instance, the 95% confidence interval is a range of values that you can be 95% confident contains the true mean of the population.

Confidence Level	Z Score
0.90 or 90%	1.645
0.95 or 95%	1.96
0.99 or 99%	2.58

The confidence interval is symmetric about $\hat{p}$, meaning the value of $\hat{p}$ lies halfway between the upper and lower values of the CI.



$$\hat{p} = \frac{lower\ value + upper\ value}{2}$$

The 95% confidence interval for the proportion of ferry tickets that are cancelled on the intended departure day is calculated from a large sample to be (0.039, 0.121).

Find the sample proportion from which this interval was constructed.

$$\hat{p} = \frac{lower\ value + upper\ value}{2}$$

$$= \frac{0.039 + 0.121}{2}$$

$$= 0.08$$

An online tutoring company is 99% sure that 20% to 30% of students prefer to use their company. What sample size was needed for this level of confidence.

99% CI $\rightarrow (0.2, 0.3)$

$$\hat{p} = \frac{0.2 + 0.3}{2}$$

$$= 0.25$$

$$n = 499.23$$

$$\therefore n = 499$$

$$\left(\hat{p} - z\sigma_{\hat{p}}, \hat{p} + z\sigma_{\hat{p}}\right) = (0.2, 0.3)$$

$$\Rightarrow 0.25 - 2.58\sqrt{\frac{0.25(1-0.25)}{n}} = 0.2$$

Margin of Error

The distance between the endpoints of the confidence interval and the sample estimate is called **the margin of error**, M.

For a 95% level of confidence,

$$M = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Use the **1-Prop z Interval** function in your CAS calculator to quickly find the lower and upper values of a CI.