Indefinite Integrals

$f(x)$ is the antiderivative of $f'(x)$, in general $\int f'(x)\ dx = f(x) + c$

- $\int a\ dx = ax + c$, where a and c are constants.
- $\int a\,x^n\ dx = \frac{ax^n}{n+1} + c, n \neq -1$

Properties of Integrals

- $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int kf(x)\,dx = k\int f(x)\,dx$, where k is a real number.

Let $f'(x) = 3x^2 - 2x$ such that $f(4) = 0$. Find $f(x)$.

$f(x) = \int 3x^2 - 2x\ dx$

$f(x) = x^3 - x^2 + c$

Given $f(4) = 0$

$4^3 - 4^2 + c = 0$
$c = -48$

$\therefore f(x) = x^3 - x^2 - 48$

Rules for Anti-Differentiation

- $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} +$ c, where $n \neq -1$
- $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \log_e(x) + c$
- $\int \frac{1}{(ax+b)}\,dx = \int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e(ax+b) + c$
- $\int e^x\,dx = e^x + c$
- $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + c$
- $\int \sin(ax)\,dx = \frac{-1}{a}\cos(ax) + c$
- $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$
- $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
- $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

Note: Anti-differentiation rules apply only to linear expressions within brackets.

For non-linear expressions use your CAS calculator to integrate.

Forgetting to include the constant '$+c$' after integrating or including 'dx' when integrating.

The anti-derivative of $\frac{3}{\sqrt{x}} - \frac{1}{(2x-2)}$ is:

$\int \frac{3}{\sqrt{x}} - \frac{1}{(2x-2)}\ dx$

$= \int 3x^{\frac{-1}{2}} - \frac{1}{(2x-2)}\ dx$

$= \frac{3}{\frac{1}{2}}x^{\frac{1}{2}} - \frac{1}{2}log_e(2x-2) + c$

$= 6x^{\frac{1}{2}} - \frac{1}{2}log_e(2x-2) + c$

$= 6\sqrt{x} - \frac{1}{2}log_e(2x-2) + c$