Normal Distribution

If X is a normal distribution, then it can be expressed as:

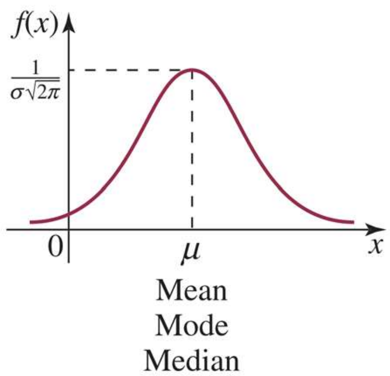
$$X \sim N(\mu, \sigma^2)$$

The equation of the normal curve is given by the PDF,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \text{ where } x \in \mathbb{R}.$$

Key Properties:

1. The distribution is symmetrical about the mean
2. The mean, mode and median are the same
3. The curve continues infinitely to the right and left of the mean, but never touches or goes below the x-axis.
4. $\Pr(a \le X \le b) = \int_a^b f(x)\ dx$
5. The area under the curve is equal to 1. $\int_{-\infty}^{\infty} f(x)\ dx = 1$



A normally distributed probability function is given by:

$$f(x) = \frac{5}{\sqrt{2\pi}} e^{-\frac{1}{2}(5(x-1))^2}, \text{ where } x \in \mathbb{R}.$$

Find the values of μ, σ and $Var(X)$.

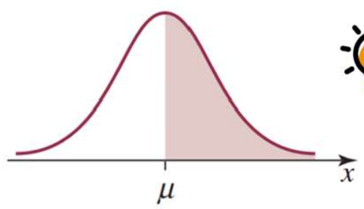
Equating $f(x)$ with the general rule

$$\frac{5}{\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}}$$

$$5 = \frac{1}{\sigma} \quad \therefore \sigma = \frac{1}{5}$$

$$\mu = 1$$

$$\Rightarrow Var(X) = \sigma^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$



$$\Pr(X > \mu) = \Pr(X < \mu) = \frac{1}{2}$$

Sketching a bell-shaped curve and shading the areas of interest can make the problem clearer.

Effect of μ and σ on Normal Graph

Changing the **mean** moves the curve left or right. (opposite the sign if using the normal curve equation)

Smaller **standard deviation:** Thinner the curve.
Larger **standard deviation**: Fatter the curve.